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Some new generalizations for $m-$convexity via new conformable fractional integral operators

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Abstract. In this paper, some new generalizations for $m-$convex functions have been given by using an integral identity via new conformable fractional integrals and some further properties. It is pointed out that special cases of our findings gave some earlier inequalities involving Riemann-Liouville fractional integrals.

1. Introduction

We will recall some definitions as follows.

Definition 1.1. A function $f : I \subseteq \mathbb{R} \to \mathbb{R}$ is said to be convex if the inequality

$$f(tx + (1-t)y) \leq tf(x) + (1-t)f(y)$$

holds for all $x, y \in I$ and $t \in [0,1]$.

$m-$convexity was defined by Toader as follow:

Definition 1.2. (See [6]) The function $f : [0,b] \to \mathbb{R}$, $b > 0$, is said to be $m-$convex, where $m \in [0,1]$,we have

$$f(tx + m(1-t)y) \leq tf(x) + m(1-t)f(y)$$

for all $x, y \in [0,b]$ and $t \in [0,1]$.

New results, generalizations and improvements for integral inequalities via different kinds of convex functions including $m-$convexity can be found in [6]-[14].

Some fractional integral operators generalize the some other fractional integrals, in special cases, as in the following integral operator. Jarad et. al. [2] has defined a new fractional integral operator. Also, they gave some properties and relations between the some other fractional integral operators, as

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Riemann-Liouville fractional integral, Hadamard fractional integrals, generalized fractional integral operators etc., with this operator.
Let $\beta\in\mathbb{C},\ Re(\beta)>0$, then the left and right sided fractional conformable integral operators has defined respectively, as follows;

$$\begin{aligned} {}^{\beta}_{a}\mathfrak{J}^{\alpha}f(x) &= \frac{1}{\Gamma(\beta)}\int_a^x\left(\frac{(x-a)^{\alpha}-(t-a)^{\alpha}}{\alpha}\right)^{\beta-1}\frac{f(t)}{(t-a)^{1-\alpha}}\mathrm{d}t;\\ {}^{\beta}\mathfrak{J}^{\alpha}_{b}f(x) &= \frac{1}{\Gamma(\beta)}\int_x^b\left(\frac{(b-x)^{\alpha}-(b-t)^{\alpha}}{\alpha}\right)^{\beta-1}\frac{f(t)}{(b-t)^{1-\alpha}}\mathrm{d}t.\end{aligned}$$

For recent results, generalizations and improvements see the papers [1]-[4].

The main purpose of this paper is to give some new results as generalizations of the previous results for $m-$convexity via new conformable fractional integral operators.

2. MAIN RESULTS

In order to prove our main theorems, we need the following lemma.

Lemma 2.1. *(See [5]) Let $f:[a,b]\to\mathbb{R}$ be a differentiable function on (a,b) with $a<b$ and $f'\in L[a,b]$.Then the following equality holds for fractional conformable integrals:*

$$\begin{aligned} &\frac{(x-a)^{\alpha\beta}+(b-x)^{\alpha\beta}}{(b-a)\alpha^{\beta}}f(x)-\frac{\Gamma(\beta+1)}{b-a}\left[{}^{\beta}_{x}\mathfrak{J}^{\alpha}f(b)+{}^{\beta}\mathfrak{J}^{\alpha}_{x}f(a)\right]\\ =\ &\frac{(x-a)^{\alpha\beta+1}}{b-a}\int_0^1\left(\frac{1-(1-t)^{\alpha}}{\alpha}\right)^{\beta}f'(tx+(1-t)a)\mathrm{d}t\\ &+\frac{(b-x)^{\alpha\beta+1}}{b-a}\int_0^1\left(\frac{1-(1-t)^{\alpha}}{\alpha}\right)^{\beta}f'(tx+(1-t)b)\mathrm{d}t\end{aligned}$$

where $\alpha,\beta>0$.

Theorem 2.1. *Let $f:[a,b]\to\mathbb{R}$ be a differentiable function on (a,b) with $a<b$ and $f'\in L[a,b]$. If $|f'|$ is $m-$convex on $[a,b]$ for some fixed $m\in(0,1]$, then the following inequality holds for fractional conformable integrals:*

$$\begin{aligned} &\left|\frac{(x-a)^{\alpha\beta}+(b-x)^{\alpha\beta}}{(b-a)\alpha^{\beta}}f(x)-\frac{\Gamma(\beta+1)}{b-a}\left[{}^{\beta}_{x}\mathfrak{J}^{\alpha}f(b)+{}^{\beta}\mathfrak{J}^{\alpha}_{x}f(a)\right]\right|\\ \le\ &\frac{m\alpha\beta}{2\left(\alpha\beta+2\right)(b-a)\,\alpha^{\beta}}\left(\left|f'\left(\frac{a}{m}\right)\right|(x-a)^{\alpha\beta+1}+\left|f'\left(\frac{b}{m}\right)\right|(b-x)^{\alpha\beta+1}\right)\\ &+\frac{|f'(x)|}{\alpha^{\beta}(b-a)}\left(\frac{\alpha^2\beta^2+3\alpha\beta}{2\left(\alpha^2\beta^2+3\alpha\beta+2\right)}\right)\left((x-a)^{\alpha\beta+1}+(b-x)^{\alpha\beta+1}\right)\end{aligned}$$

where $\alpha>0,\beta>1$.

Proof. Since $|f'|$ is $m-$convex on $[a,b]$ and by Lemma 2.1, we can write

$$\begin{aligned}
&\left|\frac{(x-a)^{\alpha\beta}+(b-x)^{\alpha\beta}}{(b-a)\alpha^{\beta}}f(x)-\frac{\Gamma(\beta+1)}{b-a}\left[{}_{x}^{\beta}\mathfrak{J}^{\alpha}f(b)+{}^{\beta}\mathfrak{J}_{x}^{\alpha}f(a)\right]\right|\\
\leq\ & \frac{(x-a)^{\alpha\beta+1}}{b-a}\int_0^1\left(\frac{1-(1-t)^{\alpha}}{\alpha}\right)^{\beta}|f'(tx+(1-t)a)|\mathrm{d}t\\
&+\frac{(b-x)^{\alpha\beta+1}}{b-a}\int_0^1\left(\frac{1-(1-t)^{\alpha}}{\alpha}\right)^{\beta}|f'(tx+(1-t)b)|\mathrm{d}t\\
=\ & \frac{(x-a)^{\alpha\beta+1}}{b-a}\int_0^1\left(\frac{1-(1-t)^{\alpha}}{\alpha}\right)^{\beta}|f'(tx+m(1-t)\frac{a}{m})|\mathrm{d}t\\
&+\frac{(b-x)^{\alpha\beta+1}}{b-a}\int_0^1\left(\frac{1-(1-t)^{\alpha}}{\alpha}\right)^{\beta}|f'(tx+m(1-t)\frac{b}{m})|\mathrm{d}t\\
\leq\ & \frac{(x-a)^{\alpha\beta+1}}{b-a}\int_0^1\left(\frac{1-(1-t)^{\alpha}}{\alpha}\right)^{\beta}\left(t\left|f'(x)\right|+m(1-t)\left|f'\left(\frac{a}{m}\right)\right|\right)\mathrm{d}t\\
&+\frac{(b-x)^{\alpha\beta+1}}{b-a}\int_0^1\left(\frac{1-(1-t)^{\alpha}}{\alpha}\right)^{\beta}\left(t\left|f'(x)\right|+m(1-t)\left|f'\left(\frac{b}{m}\right)\right|\right)\mathrm{d}t.
\end{aligned}$$

By using the fact that $|1-(1-t)^{\alpha}|^{\beta}\leq 1-|1-t|^{\alpha\beta}$ for $\alpha>0,\beta>1$, we can write

$$\begin{aligned}
&\left|\frac{(x-a)^{\alpha\beta}+(b-x)^{\alpha\beta}}{(b-a)\alpha^{\beta}}f(x)-\frac{\Gamma(\beta+1)}{b-a}\left[{}_{x}^{\beta}\mathfrak{J}^{\alpha}f(b)+{}^{\beta}\mathfrak{J}_{x}^{\alpha}f(a)\right]\right|\\
\leq\ & \frac{(x-a)^{\alpha\beta+1}}{b-a}\int_0^1\left(\frac{1-|1-t|^{\alpha\beta}}{\alpha^{\beta}}\right)\left(t\left|f'(x)\right|+m(1-t)\left|f'\left(\frac{a}{m}\right)\right|\right)\mathrm{d}t\\
&+\frac{(b-x)^{\alpha\beta+1}}{b-a}\int_0^1\left(\frac{1-|1-t|^{\alpha\beta}}{\alpha^{\beta}}\right)\left(t\left|f'(x)\right|+m(1-t)\left|f'\left(\frac{b}{m}\right)\right|\right)\mathrm{d}t
\end{aligned}$$

By computing the above integrals, the proof is completed. □

Corollary 2.1. *Under the assumptions of Theorem 1,*

1) If we choose $m=1$, we have the following inequality;

$$\begin{aligned}
&\left|\frac{(x-a)^{\alpha\beta}+(b-x)^{\alpha\beta}}{(b-a)\alpha^{\beta}}f(x)-\frac{\Gamma(\beta+1)}{b-a}\left[{}_{x}^{\beta}\mathfrak{J}^{\alpha}f(b)+{}^{\beta}\mathfrak{J}_{x}^{\alpha}f(a)\right]\right|\\
\leq\ & \frac{\alpha\beta}{2\left(\alpha\beta+2\right)\left(b-a\right)\alpha^{\beta}}\left(\left|f'\left(a\right)\right|\left(x-a\right)^{\alpha\beta+1}+\left|f'\left(b\right)\right|\left(b-x\right)^{\alpha\beta+1}\right)\\
&+\frac{f\left(x\right)}{\alpha^{\beta}(b-a)}\left(\frac{\alpha^2\beta^2+3\alpha\beta}{2\left(\alpha^2\beta^2+3\alpha\beta+2\right)}\right)\left(\left(x-a\right)^{\alpha\beta+1}+\left(b-x\right)^{\alpha\beta+1}\right).
\end{aligned}$$

2) If we choose $m=1$ and $x=\frac{a+b}{2}$, we have the following inequality;

$$\left|\frac{2^{1-\alpha\beta}(b-a)^{\alpha\beta-1}}{\alpha^{\beta}}f\left(\frac{a+b}{2}\right)-\frac{\Gamma(\beta+1)}{b-a}\left[{}_{\frac{a+b}{2}}^{\beta}\mathfrak{J}^{\alpha}f(b)+{}^{\beta}\mathfrak{J}_{\frac{a+b}{2}}^{\alpha}f(a)\right]\right|$$

$$\leq \frac{\alpha\beta\,(b-a)^{\alpha\beta}}{2^{\alpha\beta+2}\,(\alpha\beta+2)\,\alpha^{\beta}}\left(\left|f'(a)\right|+\left|f'(b)\right|\right)$$
$$+\frac{(b-a)^{\alpha\beta}}{\alpha^{\beta}2^{\alpha\beta+1}}\left(\frac{\alpha^2\beta^2+3\alpha\beta}{(\alpha^2\beta^2+3\alpha\beta+2)}\right)\left|f'\left(\frac{a+b}{2}\right)\right|.$$

Theorem 2.2. *Let $f:[a,b]\to\mathbb{R}$ be a differentiable function on (a,b) with $a<b$ and $f'\in L[a,b]$. If $|f'|^q$ is $m-$convex on $[a,b]$ for some fixed $m\in(0,1]$ and $p,q>1$, then the following inequality holds for fractional conformable integrals:*

$$\left|\frac{(x-a)^{\alpha\beta}+(b-x)^{\alpha\beta}}{(b-a)\alpha^{\beta}}f(x)-\frac{\Gamma(\beta+1)}{b-a}\left[{}^{\beta}_{x}\mathfrak{J}^{\alpha}f(b)+{}^{\beta}\,\mathfrak{J}^{\alpha}_{x}f(a)\right]\right|$$
$$\leq \frac{(x-a)^{\alpha\beta+1}}{b-a}\left(\frac{B\left(\beta p+1,\frac{1}{\alpha}\right)}{\alpha^{\beta+1}}\right)^{\frac{1}{p}}\left(\frac{|f'(x)|^q+m\left|f'\left(\frac{a}{m}\right)\right|^q}{2}\right)^{\frac{1}{q}}$$
$$+\frac{(b-x)^{\alpha\beta+1}}{b-a}\left(\frac{B\left(\beta p+1,\frac{1}{\alpha}\right)}{\alpha^{\beta+1}}\right)^{\frac{1}{p}}\left(\frac{|f'(x)|^q+m\left|f'\left(\frac{b}{m}\right)\right|^q}{2}\right)^{\frac{1}{q}}.$$

where $\frac{1}{p}+\frac{1}{q}=1$, $\alpha,\beta>0$, $B(x,y)$ is Euler Beta function.

Proof. From Lemma 2.1 and using the Hölder inequality, we have

$$\left|\frac{(x-a)^{\alpha\beta}+(b-x)^{\alpha\beta}}{(b-a)\alpha^{\beta}}f(x)-\frac{\Gamma(\beta+1)}{b-a}\left[{}^{\beta}_{x}\mathfrak{J}^{\alpha}f(b)+{}^{\beta}\,\mathfrak{J}^{\alpha}_{x}f(a)\right]\right|$$
$$\leq \frac{(x-a)^{\alpha\beta+1}}{b-a}\left(\int_0^1\left(\frac{1-(1-t)^{\alpha}}{\alpha}\right)^{\beta p}\mathrm{d}t\right)^{\frac{1}{p}}\cdot$$
$$\cdot\left(\int_0^1\left(t\left|f'(x)\right|^q+m(1-t)\left|f'\left(\frac{a}{m}\right)\right|^q\right)\mathrm{d}t\right)^{\frac{1}{q}}$$
$$+\frac{(b-x)^{\alpha\beta+1}}{b-a}\left(\int_0^1\left(\frac{1-(1-t)^{\alpha}}{\alpha}\right)^{\beta p}dt\right)^{\frac{1}{p}}\cdot$$
$$\cdots\left(\int_0^1\left(t\left|f'(x)\right|^q+m(1-t)\left|f'\left(\frac{b}{m}\right)\right|^q\right)\mathrm{d}t\right)^{\frac{1}{q}}.$$

If the above integrals are calculated, we obtain

$$\left|\frac{(x-a)^{\alpha\beta}+(b-x)^{\alpha\beta}}{(b-a)\alpha^{\beta}}f(x)-\frac{\Gamma(\beta+1)}{b-a}\left[{}^{\beta}_{x}\mathfrak{J}^{\alpha}f(b)+{}^{\beta}\,\mathfrak{J}^{\alpha}_{x}f(a)\right]\right|$$
$$\leq \frac{(x-a)^{\alpha\beta+1}}{b-a}\left(\frac{B\left(\beta p+1,\frac{1}{\alpha}\right)}{\alpha^{\beta+1}}\right)^{\frac{1}{p}}\left(\frac{|f'(x)|^q+m\left|f'\left(\frac{a}{m}\right)\right|^q}{2}\right)^{\frac{1}{q}}$$

$$+\frac{(b-x)^{\alpha\beta+1}}{b-a}\left(\frac{B\left(\beta p+1,\frac{1}{\alpha}\right)}{\alpha^{\beta+1}}\right)^{\frac{1}{p}}\left(\frac{|f'(x)|^q+m\left|f'\left(\frac{b}{m}\right)\right|^q}{2}\right)^{\frac{1}{q}}.$$

The proof is completed. □

Corollary 2.2. *Under the assumptions of Theorem 2,*

1) If we choose $m=1$, we have the following inequality;

$$\left|\frac{(x-a)^{\alpha\beta}+(b-x)^{\alpha\beta}}{(b-a)\alpha^{\beta}}f(x)-\frac{\Gamma(\beta+1)}{b-a}\left[{}^{\beta}_{x}\mathfrak{J}^{\alpha}f(b)+{}^{\beta}\mathfrak{J}^{\alpha}_{x}f(a)\right]\right|$$

$$\leq \frac{(x-a)^{\alpha\beta+1}}{b-a}\left(\frac{B\left(\beta p+1,\frac{1}{\alpha}\right)}{\alpha^{\beta+1}}\right)^{\frac{1}{p}}\left(\frac{|f'(x)|^q+|f'(a)|^q}{2}\right)^{\frac{1}{q}}$$

$$+\frac{(b-x)^{\alpha\beta+1}}{b-a}\left(\frac{B\left(\beta p+1,\frac{1}{\alpha}\right)}{\alpha^{\beta+1}}\right)^{\frac{1}{p}}\left(\frac{|f'(x)|^q+|f'(b)|^q}{2}\right)^{\frac{1}{q}}.$$

2) If we choose $m=1$ and $x=\frac{a+b}{2}$, we have the following inequality;

$$\left|\frac{2^{1-\alpha\beta}(b-a)^{\alpha\beta-1}}{\alpha^{\beta}}f\left(\frac{a+b}{2}\right)-\frac{\Gamma(\beta+1)}{b-a}\left[{}^{\beta}_{\frac{a+b}{2}}\mathfrak{J}^{\alpha}f(b)+{}^{\beta}\mathfrak{J}^{\alpha}_{\frac{a+b}{2}}f(a)\right]\right|$$

$$\leq \frac{(b-a)^{\alpha\beta}}{2^{\alpha\beta+1}}\left(\frac{B\left(\beta p+1,\frac{1}{\alpha}\right)}{\alpha^{\beta+1}}\right)^{\frac{1}{p}}\left(\frac{\left|f'\left(\frac{a+b}{2}\right)\right|^q+|f'(a)|^q}{2}\right)^{\frac{1}{q}}$$

$$+\frac{(b-a)^{\alpha\beta}}{2^{\alpha\beta+1}}\left(\frac{B\left(\beta p+1,\frac{1}{\alpha}\right)}{\alpha^{\beta+1}}\right)^{\frac{1}{p}}\left(\frac{\left|f'\left(\frac{a+b}{2}\right)\right|^q+|f'(b)|^q}{2}\right)^{\frac{1}{q}}.$$

Remark 2.1. If we choose $|f'|<M$ in Corollary 2 (i), we obtain Theorem 2.2 of [5].

Theorem 2.3. *Let $f:[a,b]\to\mathbb{R}$ be a differentiable function on (a,b) with $a<b$ and $f'\in L[a,b]$. If $|f'|^q$ is $m-$convex on $[a,b]$ for some fixed $m\in(0,1]$ and $q\geq 1$, then the following inequality holds for fractional conformable integrals:*

$$\left|\frac{(x-a)^{\alpha\beta}+(b-x)^{\alpha\beta}}{(b-a)\alpha^{\beta}}f(x)-\frac{\Gamma(\beta+1)}{b-a}\left[{}^{\beta}_{x}\mathfrak{J}^{\alpha}f(b)+{}^{\beta}\mathfrak{J}^{\alpha}_{x}f(a)\right]\right|$$

$$\leq \frac{(x-a)^{\alpha\beta+1}}{b-a}\left(\frac{B\left(\frac{1}{\alpha},\beta+1\right)}{\alpha^{\beta+1}}\right)^{1-\frac{1}{q}}$$

$$\times\left(\frac{|f'(x)|^q}{\alpha^{\beta}}\left(\frac{1}{2}-B(2,\alpha\beta+1)\right)+\frac{m\left|f'\left(\frac{a}{m}\right)\right|^q}{\alpha^{\beta}}\left(\frac{\alpha\beta}{2(\alpha\beta+2)}\right)\right)^{\frac{1}{q}}$$

$$+\frac{(b-x)^{\alpha\beta+1}}{b-a}\left(\frac{B\left(\frac{1}{\alpha},\beta+1\right)}{\alpha^{\beta+1}}\right)^{1-\frac{1}{q}}$$

$$\times\left(\frac{|f'(x)|^q}{\alpha^\beta}\left(\frac{1}{2}-B\left(2,\alpha\beta+1\right)\right)+\frac{m\left|f'\left(\frac{b}{m}\right)\right|^q}{\alpha^\beta}\left(\frac{\alpha\beta}{2\left(\alpha\beta+2\right)}\right)\right)^{\frac{1}{q}}.$$

where $\alpha>0,\beta>1$, $B(x,y)$ is Euler Beta function.

Proof. Since $|f'|^q$ is $m-$convex on $[a,b]$ for $q\geq 1$, from Lemma 2.1 and the well known power mean integral inequality, we can write

$$\begin{aligned}
&\left|\frac{(x-a)^{\alpha\beta}+(b-x)^{\alpha\beta}}{(b-a)\alpha^\beta}f(x)-\frac{\Gamma(\beta+1)}{b-a}\left[{}_x^\beta\mathfrak{J}^\alpha f(b)+{}^\beta\,\mathfrak{J}_x^\alpha f(a)\right]\right| \\
\leq\ & \frac{(x-a)^{\alpha\beta+1}}{b-a}\left(\int_0^1\left(\frac{1-(1-t)^\alpha}{\alpha}\right)^\beta \mathrm{d}t\right)^{1-\frac{1}{q}} \\
&\times\left(\int_0^1\left(\frac{1-(1-t)^\alpha}{\alpha}\right)^\beta\left(t\left|f'(x)\right|^q+m(1-t)\left|f'\left(\frac{a}{m}\right)\right|^q\right)\mathrm{d}t\right)^{\frac{1}{q}} \\
&+\frac{(b-x)^{\alpha\beta+1}}{b-a}\left(\int_0^1\left(\frac{1-(1-t)^\alpha}{\alpha}\right)^\beta \mathrm{d}t\right)^{1-\frac{1}{q}} \\
&\times\left(\int_0^1\left(\frac{1-(1-t)^\alpha}{\alpha}\right)^\beta\left(t\left|f'(x)\right|^q+m(1-t)\left|f'\left(\frac{b}{m}\right)\right|^q\right)\mathrm{d}t\right)^{\frac{1}{q}}.
\end{aligned}$$

By a simple computation and by using the fact that $|1-(1-t)^\alpha|^\beta\leq 1-|1-t|^{\alpha\beta}$ for $\alpha>0,\beta>1$, we get

$$\begin{aligned}
&\left|\frac{(x-a)^{\alpha\beta}+(b-x)^{\alpha\beta}}{(b-a)\alpha^\beta}f(x)-\frac{\Gamma(\beta+1)}{b-a}\left[{}_x^\beta\mathfrak{J}^\alpha f(b)+{}^\beta\,\mathfrak{J}_x^\alpha f(a)\right]\right| \\
\leq\ & \frac{(x-a)^{\alpha\beta+1}}{b-a}\left(\frac{B\left(\frac{1}{\alpha},\beta+1\right)}{\alpha^{\beta+1}}\right)^{1-\frac{1}{q}} \\
&\times\left(\frac{|f'(x)|^q}{\alpha^\beta}\left(\frac{1}{2}-B\left(2,\alpha\beta+1\right)\right)+\frac{m\left|f'\left(\frac{a}{m}\right)\right|^q}{\alpha^\beta}\left(\frac{\alpha\beta}{2\left(\alpha\beta+2\right)}\right)\right)^{\frac{1}{q}} \\
&+\frac{(b-x)^{\alpha\beta+1}}{b-a}\left(\frac{B\left(\frac{1}{\alpha},\beta+1\right)}{\alpha^{\beta+1}}\right)^{1-\frac{1}{q}} \\
&\times\left(\frac{|f'(x)|^q}{\alpha^\beta}\left(\frac{1}{2}-B\left(2,\alpha\beta+1\right)\right)+\frac{m\left|f'\left(\frac{b}{m}\right)\right|^q}{\alpha^\beta}\left(\frac{\alpha\beta}{2\left(\alpha\beta+2\right)}\right)\right)^{\frac{1}{q}}.
\end{aligned}$$

the result is obtained and the proof is completed. □

Corollary 2.3. *Under the assumptions of Theorem 3:*

1) If we choose $m=1$, we have the following inequality:

$$\begin{aligned}
&\left|\frac{(x-a)^{\alpha\beta}+(b-x)^{\alpha\beta}}{(b-a)\alpha^{\beta}}f(x)-\frac{\Gamma(\beta+1)}{b-a}\left[{}_{x}^{\beta}\mathfrak{J}^{\alpha}f(b)+^{\beta}\mathfrak{J}_{x}^{\alpha}f(a)\right]\right| \\
\leq\;& \frac{(x-a)^{\alpha\beta+1}}{b-a}\left(\frac{B\left(\frac{1}{\alpha},\beta+1\right)}{\alpha^{\beta+1}}\right)^{1-\frac{1}{q}} \\
&\times\left(\frac{|f'(x)|^{q}}{\alpha^{\beta}}\left(\frac{1}{2}-B\left(2,\alpha\beta+1\right)\right)+\frac{|f'\left(a\right)|^{q}}{\alpha^{\beta}}\left(\frac{\alpha\beta}{2\left(\alpha\beta+2\right)}\right)\right)^{\frac{1}{q}} \\
&+\frac{(b-x)^{\alpha\beta+1}}{b-a}\left(\frac{B\left(\frac{1}{\alpha},\beta+1\right)}{\alpha^{\beta+1}}\right)^{1-\frac{1}{q}} \\
&\times\left(\frac{|f'(x)|^{q}}{\alpha^{\beta}}\left(\frac{1}{2}-B\left(2,\alpha\beta+1\right)\right)+\frac{|f'\left(b\right)|^{q}}{\alpha^{\beta}}\left(\frac{\alpha\beta}{2\left(\alpha\beta+2\right)}\right)\right)^{\frac{1}{q}}.
\end{aligned}$$

2) If we choose $m=1$ and $x=\frac{a+b}{2}$, we have the following inequality:

$$\begin{aligned}
&\left|\frac{2^{1-\alpha\beta}(b-a)^{\alpha\beta-1}}{\alpha^{\beta}}f\left(\frac{a+b}{2}\right)-\frac{\Gamma(\beta+1)}{b-a}\left[{}_{\frac{a+b}{2}}^{\beta}\mathfrak{J}^{\alpha}f(b)+^{\beta}\mathfrak{J}_{\frac{a+b}{2}}^{\alpha}f(a)\right]\right| \\
\leq\;& \frac{(b-a)^{\alpha\beta}}{2^{\alpha\beta+1}}\left(\frac{B\left(\frac{1}{\alpha},\beta+1\right)}{\alpha^{\beta+1}}\right)^{1-\frac{1}{q}} \\
&\times\left(\frac{\left|f'(\frac{a+b}{2})\right|^{q}}{\alpha^{\beta}}\left(\frac{1}{2}-B\left(2,\alpha\beta+1\right)\right)+\frac{|f'\left(a\right)|^{q}}{\alpha^{\beta}}\left(\frac{\alpha\beta}{2\left(\alpha\beta+2\right)}\right)\right)^{\frac{1}{q}} \\
&+\frac{(b-a)^{\alpha\beta}}{2^{\alpha\beta+1}}\left(\frac{B\left(\frac{1}{\alpha},\beta+1\right)}{\alpha^{\beta+1}}\right)^{1-\frac{1}{q}} \\
&\times\left(\frac{\left|f'(\frac{a+b}{2})\right|^{q}}{\alpha^{\beta}}\left(\frac{1}{2}-B\left(2,\alpha\beta+1\right)\right)+\frac{|f'\left(b\right)|^{q}}{\alpha^{\beta}}\left(\frac{\alpha\beta}{2\left(\alpha\beta+2\right)}\right)\right)^{\frac{1}{q}}.
\end{aligned}$$

Remark 2.2. If we choose $|f'|<M$ in Corollary 3 (i), we obtain Theorem 2.3 of [5].

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